2.4, 2.10
Applications of the First Law to ideal gases

What we know

Equation of state

\[ pv = RT \]

The first law

\[ du = dq + dw \]
\[ dh = dq + vdp \]

Internal energy is function only of temperature

\[ du = c_vdT \]
\[ dh = c_pdT \]

Specific heats are related

\[ c_p - c_v = R \]

"expansion work"

\[ dw = -pdu \]
The first law of thermodynamics for an ideal gas

\[ du = c_v dT \quad dh = c_p dT \quad c_p - c_v = R \quad dw = -pdv \]

\[ du = dq + dw \quad c_v dT = dq - pdv \]

\[ dh = dq + vdp \quad c_p dT = dq + vdp \]
An *isothermal process* in one in which the initial and final temperatures are the same.

\[ dT = 0 \]

An *adiabatic process* in one in which no heat is exchanged between the system and its surroundings.

\[ dq = 0 \]

Isothermal processes are not necessarily adiabatic.
Reversible Changes

- A **reversible change** is one that can be reversed by an infinitesimal modification of a variable.

- In a reversible expansion or compression

\[ P_{env} = P_{gas} \]
Let's consider an isothermal expansion: \( dT = 0 \)

\[
du = c_v \, dT
\]

Because internal energy is a function only of temperature, the internal energy of the gas is unchanged.

\[
c_v \, dT = dq - pdv
\]

\[
dq = pdv
\]

The first law of thermodynamics for an isothermal expansion:

\[
dq = pdv
\]
If we want to integrate

\[ dq = pdv \]

only expansion work"

\[ dw = -pdv \]

\[ \int dw \neq 0 \]

What do we need????

**Path!!!**

Lets consider the path of an isothermal reversible expansion

Equation of state is satisfied during all the stages of the expansion

\[ pv = RT \]

**Isothermal Expansion**
If we want to integrate

\[ dq = pdv \]

\[ pv = RT \]

\[ p = \frac{RT}{v} \]

\[ dq = RT \frac{dv}{v} \]

\[ \Delta q = RT \int_{v_1}^{v_2} \frac{dv}{v} \]

\[ \Delta q = RT \ln \left( \frac{v_2}{v_1} \right) \]

\[ v = \frac{RT}{p} \]

\[ \Delta q = RT \ln \left( \frac{p_1}{p_2} \right) \]

\[ \Delta q = RT \ln \left( \frac{v_2}{v_1} \right) = RT \ln \left( \frac{p_1}{p_2} \right) \]

isothermal expansion
The amount of heat required to raise the temperature of the gas from $T_1$ to $T_2$ at constant volume

$$\Delta q = c_v (T_2 - T_1)$$
\[ du = c_v \, dT \quad dh = c_p \, dT \quad c_p - c_v = R \quad dw = -pdv \]

**Constant pressure process**

\[ dp = 0 \]

\[ dh = dq + vdp \]

\[ dh = dq \]

\[ dh = c_p \, dT \]

\[ dq = c_p \, dT \]

The amount of heat required to raise the temperature of the gas from \( T_1 \) to \( T_2 \) at constant pressure

\[ \Delta q = c_p \left( T_2 - T_1 \right) \]
Adiabatic process

Process in which NO HEAT is exchange between the system and its environment

\[ dq = 0 \]

\[ du = dq + dw \]

Fist law for a reversible adiabatic process

\[ du = dw \]

An adiabatic compression increases the internal energy of the system
The first law of thermodynamics for an ideal gas

\[ c_v dT = dq - pdv \]

\[ c_p dT = dq + vdp \]

Adiabatic process

\[ dq = 0 \]

\[ c_v dT = -pdv \]

\[ c_p dT = vdp \]

Fist law for adiabatic expansion
Poisson’s equations

\[ dq = 0 \]

Let's consider a reversible adiabatic \textbf{EXPANSION} for an ideal gas

\[ c_v \, dT = -pdv \]
\[ pv = RT \]
\[ p = \frac{RT}{v} \]

\[ c_v \, dT = -\frac{RT}{v} \, dv \]
\[ c_v \, \frac{dT}{T} = -R \, \frac{dv}{v} \]

Let's assume \( c_v \) constant. There is a final and initial state

\[ c_v \int_{T_1}^{T_2} \frac{dT}{T} = -R \int_{v_1}^{v_2} \frac{dv}{v} \]
\[ c_v \int_{T_1}^{T_2} \frac{dT}{T} = -R \int_{v_1}^{v_2} \frac{dv}{v} \]

\[ c_v \ln\left(\frac{T_2}{T_1}\right) = -R \ln\left(\frac{v_2}{v_1}\right) \]

During an adiabatic expansion of a gas, the temperature decreases.

\[ \left(\frac{T_2}{T_1}\right) = \left(\frac{v_1}{v_2}\right)^{\frac{R}{c_v}} \]

Reverse process \rightarrow adiabatic Compression

Work is done on the gas and the temperature increases.
\[
\left( \frac{T_2}{T_1} \right) = \left( \frac{v_1}{v_2} \right)^{R/c_v}
\]

\[
\left( \frac{p_2}{p_1} \right) = \left( \frac{v_1}{v_2} \right)^{c_p/c_v}
\]

\[
\left( \frac{T_2}{T_1} \right) = \left( \frac{p_2}{p_1} \right)^{R/c_p}
\]

\[
pv = RT
\]
A given pressure decrease produces a smaller volume increase in the adiabatic case relative to the isothermal case. Temperature also decreases during the adiabatic expansion.

\[ pv = RT \]
Dry Adiabatic Processes in the Atmosphere

For reversible adiabatic processes for an ideal gas:

\[
\frac{T_0}{T} = \left( \frac{p_0}{p} \right)^{\frac{R}{c_p}}
\]

Lifting Processes

Orographic Lifting

Frontal Lifting

Low-level convergence

Vertical Mixing

\[dq = 0\]

\[p \downarrow\]

\[T \downarrow\]
A dry adiabatic process as long CONDENSATION DOES NOT OCCUR!!
If for $p_0 = 1000 \text{ mb}$ the temperature is $\theta$,

$$\frac{T_0}{T} = \left( \frac{p_0}{p} \right)^{\frac{R}{c_p}} \quad \frac{\theta}{T} = \left( \frac{p_0}{p} \right)^{\frac{R}{c_p}} \quad \rightarrow \quad \theta = T \left( \frac{p_0}{p} \right)^{\frac{R}{c_p}}$$

Where $\frac{R}{c_p}$ for dry air is

$$\frac{R}{c_p} = \frac{R}{c_v + R} = \frac{R}{\frac{5}{2}R + R} = \frac{2}{7} = 0.286$$

\[\theta\] Potential Temperature

Temperature a parcel of gas (e.g. dry air) would have if compressed (or expanded) in an reversible adiabatic process from a state, $p$ and $T$, to a a pressure of $p_0 = 1000 \text{ mb}$

\[\theta\] Also a state variable, invariant during a reversible adiabatic process: **Conservative quantity!!**
Consider a Temperature profile with

\[ \Gamma = 6^\circ \text{C km}^{-1} \]

\[ \Gamma = -\frac{dT}{dz} \]

\[ p = 1000 \text{ mb} \quad \Rightarrow \quad \theta = T \]

For \( p < 1000 \text{ mb} \quad \Rightarrow \quad \theta > T \quad \text{Adiabatic compression to lower the parcel} \]

\[ p > 1000 \text{ mb} \quad \Rightarrow \quad \theta < T \quad \text{Adiabatic expansion to rise the parcel} \]
Considering Water vapor

\[ \theta = T \left( \frac{p_0}{p} \right)^{\frac{R}{c_p}} \]  

\[ \frac{R}{c_p} \]  

Considering water vapor

Specific heat for moist air

\[ c_p = (1 - q_v)c_{pd} + q_v c_{pv} \approx c_{pd} (1 + 0.87q_v) \]

\[ d \]  For dry air  \[ v \]  For water vapor  \[ q_v \]  Specific humidity

And knowing (from Ch. 1)

\[ R = R_d (1 + 0.608q_v) \]

\[ \frac{R}{c_p} = \frac{R_d}{c_{pd}} \left( \frac{1 + 0.608q_v}{1 + 0.87q_v} \right) \approx \frac{R_d}{c_{pd}} (1 - 0.26q_v) \]

\[ \theta = T \left( \frac{p_0}{p} \right)^{\frac{R_d (1 - 0.26q_v)}{c_{pd}}} \]  

\[ \text{Potential Temperature for Moist Air} \]
The difference between dry-air and moist air potential temperature is generally less than 0.1 degree.

Adiabatic expansion or compression of moisture air can be treated as if it were dry air.

**Important:** $\theta$ is not conserved if a change of phase occurs:
- Liquid to vapor
- Solid to Liquid
Virtual Temperature

\[ T_v \]

the temperature a parcel which contains no moisture would have to equal the density of a parcel at a specific temperature and humidity.

Virtual Potential Temperature

\[ \theta_v \]

temperature a parcel at a specific pressure level and virtual temperature would have if it were lowered or raised to 1000 mb.

Virtual Potential Temperature

Neglect the dependence on water vapor from the exponent, and replace temperature by virtual temperature

\[
\theta = T \left( \frac{p_0}{p} \right)^{R_d (1-0.26q_v) / c_{pd}} \Rightarrow \theta_v = T_v \left( \frac{p_0}{p} \right)^{R_d / c_{pd}}
\]
Adiabatic Ascent of a Parcel

\[ T \downarrow \quad \theta \quad \text{constant} \]

Rate of decrease of temperature with height  
(first law, enthalpy form)

First law for adiabatic process

\[ c_p \, dT = vdp \]

If there are no large vertical accelerations  
(hydrostatic relation applies)

\[ -gdz = vdp \quad \rightarrow \quad c_p \, dT = -gdz \]

\[ \Gamma = -\frac{dT}{dz} \quad \Gamma_d = \frac{g}{c_{pd}} \approx 9.8^0 \text{C km}^{-1} \quad \text{Lapse rate for dry air} \]

For moist air

\[ \Gamma = \frac{g}{c_{pd}(1+0.87q_v)} \]