

2.4, 2.10

## Applications of the First Law to ideal gases

What we know

Equation of state

$$pv = RT$$

The first law

$$du = dq + dw$$

$$dh = dq + vdp$$

Internal energy is function  
only of temperature

$$du = c_v dT$$

$$dh = c_p dT$$

Specific heats are related

$$c_p - c_v = R$$

"expansion work"

$$dw = -pdv$$

$$du = c_v dT$$

$$dh = c_p dT$$

$$c_p - c_v = R$$

$$dw = -pdv$$

The first law of thermodynamics for an ideal gas

$$du = dq + dw$$

$$c_v dT = dq - pdv$$

$$dh = dq + vdp$$

$$c_p dT = dq + vdp$$

## Isothermal vs. Adiabatic

An *isothermal process* is one in which the initial and final temperatures are the same.

$$dT = 0$$

An *adiabatic process* is one in which no heat is exchanged between the system and its surroundings.

$$dq = 0$$

Isothermal processes are not necessarily adiabatic.

## Reversible Changes

- A ***reversible change*** is one that can be reversed by an infinitesimal modification of a variable.
- In a reversible expansion or compression

$$P_{env} = P_{gas}$$

Lets consider an isothermal expansion

$$dT = 0$$

$$du = c_v dT$$

Because Internal energy is function only of temperature, the internal energy of the gas is unchanged

$$c_v dT = dq - pdv$$



$$dq = pdv$$

The first law of thermodynamics for an isothermal expansion

$$dq = pdv$$

isothermal expansion

If we want to integrate

$$dq = pdv$$

only expansion work"

$$dw = -pdv$$

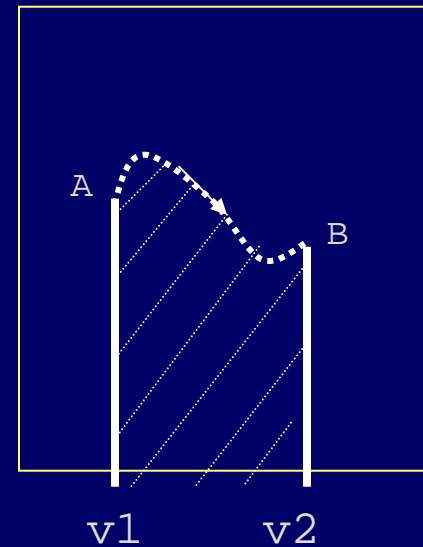
$$\oint dw \neq 0$$

What do we need????  $\longrightarrow$  Path!!!

Lets consider the path of an isothermal reversible expansion

Equation of state is satisfied during all the stages of the expansion

$$pv = RT$$



isothermal expansion

If we want to integrate

$$dq = p dv$$

$$pv = RT$$

$$p = \frac{RT}{v}$$

$$dq = RT \frac{dv}{v}$$

$$\Delta q = RT \int_{v_1}^{v_2} \frac{dv}{v}$$

$$\Delta q = RT \ln \left( \frac{v_2}{v_1} \right)$$

$$v = \frac{RT}{p}$$

$$\Delta q = RT \ln \left( \frac{p_1}{p_2} \right)$$

$$\Delta q = RT \ln \left( \frac{v_2}{v_1} \right) = RT \ln \left( \frac{p_1}{p_2} \right)$$

isothermal expansion



$$du = c_v dT$$

$$dh = c_p dT$$

$$c_p - c_v = R$$

$$dw = -pdv$$

Constant volume process

$$dv = 0$$

$$du = dq + dw$$

$$du = dq - pdv$$

$$du = dq$$

$$du = c_v dT$$

$$dq = c_v dT$$

The amount of heat required to raise the temperature of the gas from  $T_1$  to  $T_2$  at constant volume

$$\Delta q = c_v (T_2 - T_1)$$

Constant volume

$$du = c_v dT$$

$$dh = c_p dT$$

$$c_p - c_v = R$$

$$dw = -p dv$$

Constant pressure process

$$dp = 0$$

$$dh = dq + v dp$$

$$dh = dq$$

$$dh = c_p dT$$

$$dq = c_p dT$$

The amount of heat required to raise the temperature of the gas from  $T_1$  to  $T_2$  at constant pressure

$$\Delta q = c_p (T_2 - T_1)$$

Constant pressure

Adiabatic process

Process in which NO HEAT is exchange between the system and its environment

$$dq = 0$$

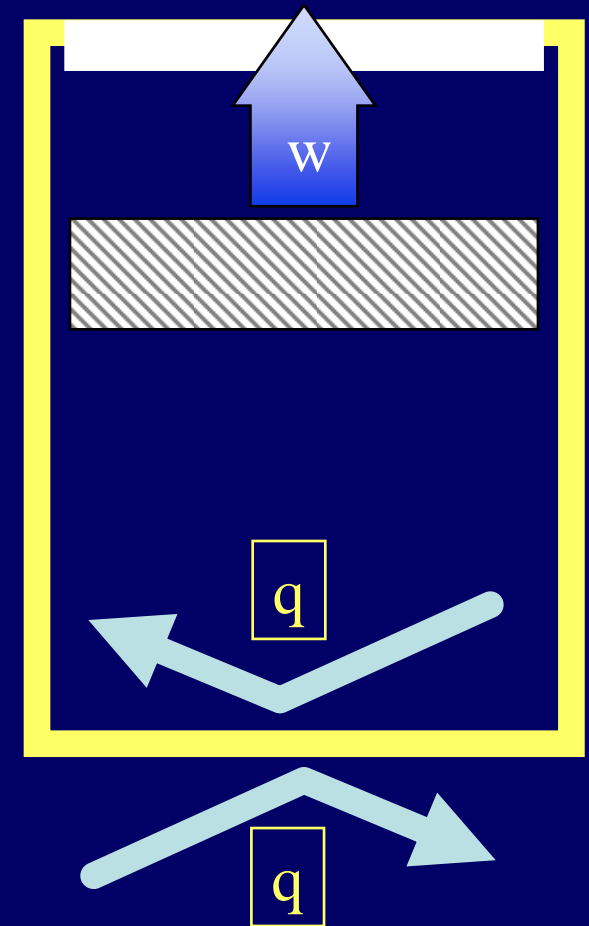
$$du = dq + dw$$

$$du = dw$$

First law for a reversible adiabatic process

$$du = -pdv$$

An adiabatic compression increases the internal energy of the system



Adiabatic process

The first law of thermodynamics for an ideal gas

$$c_v dT = dq - pdv$$

$$c_p dT = dq + vdp$$

Adiabatic process

$$dq = 0$$

$$c_v dT = -pdv$$

$$c_p dT = vdp$$

First law for adiabatic expansion

Adiabatic process

Poisson's equations

$$dq = 0$$

Lets consider a reversible adiabatic **EXPANSION** for an ideal gas

$$c_v dT = -p dv$$

$$pv = RT$$

$$p = \frac{RT}{v}$$

$$c_v dT = -\frac{RT}{v} dv$$

$$c_v \frac{dT}{T} = -R \frac{dv}{v}$$

Lets assume  $c_v$   
constant

There is a final and initial state

$$c_v \int_{T_1}^{T_2} \frac{dT}{T} = -R \int_{v_1}^{v_2} \frac{dv}{v}$$

## Poisson's equations

$$c_v \int_{T_1}^{T_2} \frac{dT}{T} = -R \int_{v_1}^{v_2} \frac{dv}{v}$$

$$c_v \ln\left(\frac{T_2}{T_1}\right) = -R \ln\left(\frac{v_2}{v_1}\right)$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{v_1}{v_2}\right)^{R/c_v}$$

During an adiabatic expansion of a gas, the temperature decreases

Reverse process



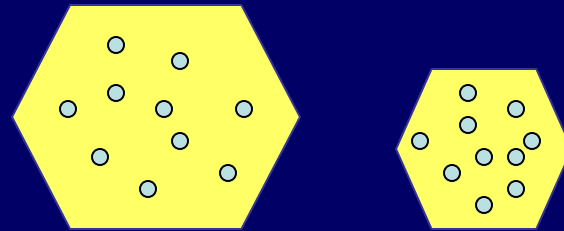
**adiabatic Compression**

Work is done on the gas and the temperature increases

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{v_1}{v_2}\right)^{R/c_v}$$

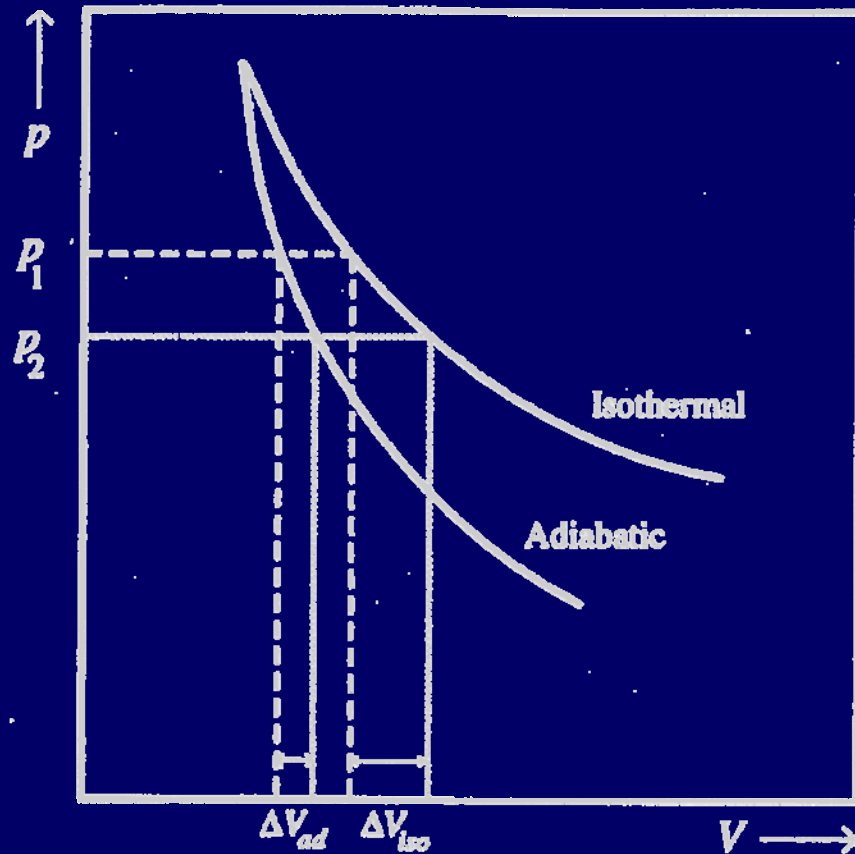
$$\left(\frac{p_2}{p_1}\right) = \left(\frac{v_1}{v_2}\right)^{c_p/c_v}$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{p_2}{p_1}\right)^{R/c_p}$$



$$pv = RT$$

isothermal expansion Vs reversible adiabatic expansion



P, v diagram

$$pv = RT$$

A given pressure decrease produces a smaller volume increase in the adiabatic case relative to the isothermal case

Temperature also decreases during the adiabatic expansion



# Dry Adiabatic Processes in the Atmosphere

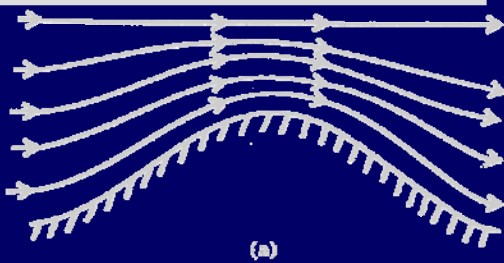
$$dq = 0$$

$$\frac{T_0}{T} = \left( \frac{p_0}{p} \right)^{R/c_p}$$

For reversible adiabatic processes for an ideal gas

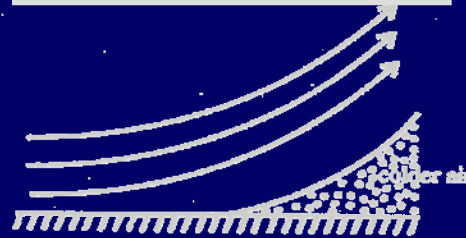
## Lifting Processes

### Orographic Lifting

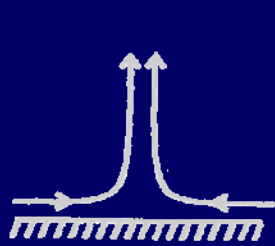
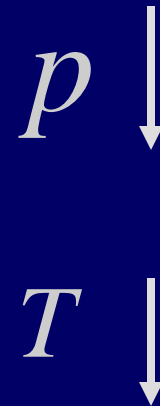


(a)

### Frontal Lifting



(b)



(c)



(d)

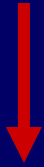


(e)

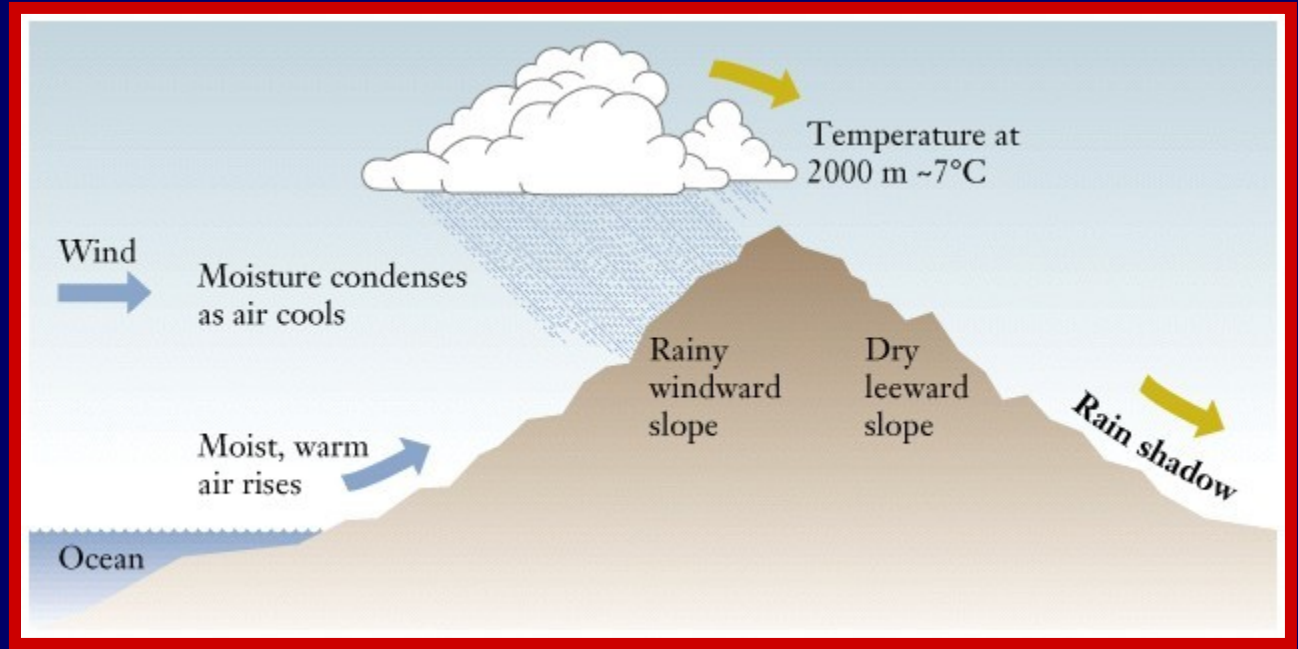
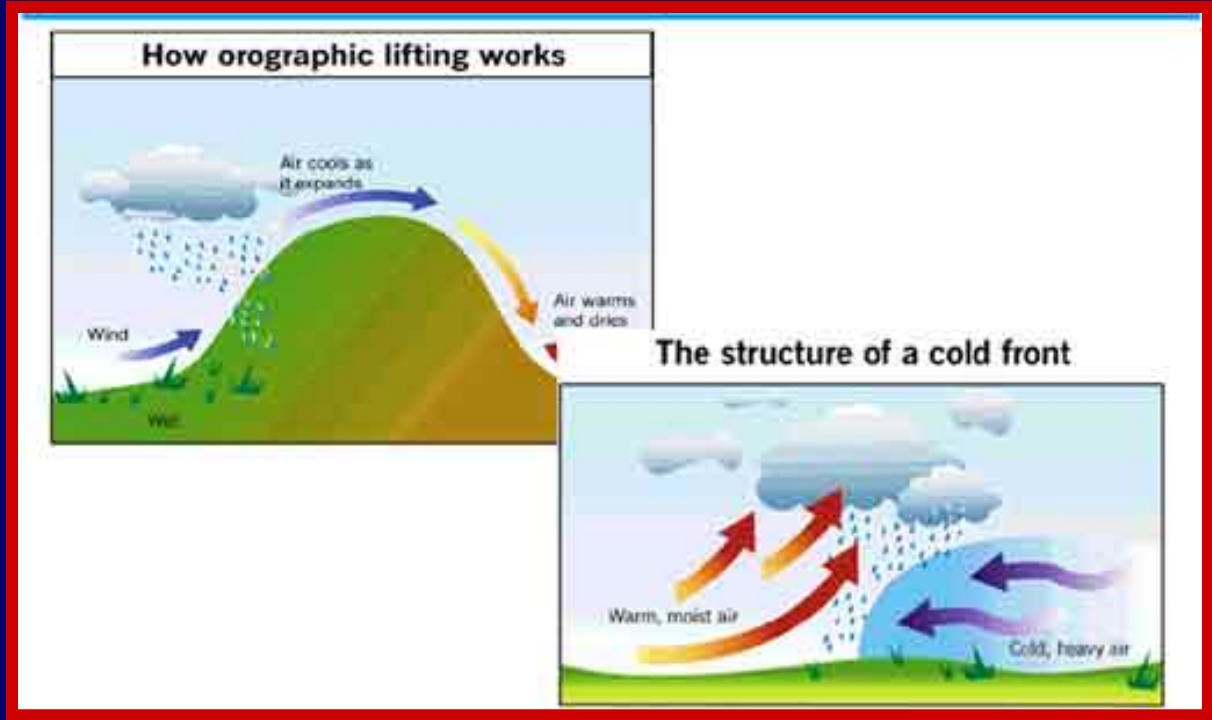
Low-level convergence

Vertical Mixing

Lifting:



A dry  
adiabatic  
process as  
long  
CONDENSATION  
DOES NOT  
OCCUR!!



If for  $p_0 = 1000$  mb the temperature is  $\theta$ ,

$$\frac{T_0}{T} = \left( \frac{p_0}{p} \right)^{R/c_p} \quad \frac{\theta}{T} = \left( \frac{p_0}{p} \right)^{R/c_p} \quad \longrightarrow \quad \theta = T \left( \frac{p_0}{p} \right)^{R/c_p}$$

Where  $\frac{R}{c_p}$  for dry air is  $\frac{R}{c_p} = \frac{R}{c_v + R} = \frac{R}{\frac{5}{2}R + R} = \frac{2}{7} = 0.286$

$\theta$   $\longrightarrow$  Potential Temperature

Temperature a parcel of gas (e.g. dry air) would have if compressed (or expanded) in an reversible adiabatic process from a state,  $p$  and  $T$ , to a a pressure of  $p_0 = 1000$  mb

$\theta$  Also a state variable, invariant during a reversible adiabatic process:

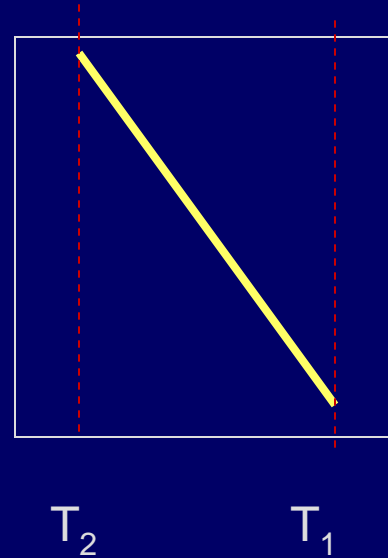
**Conservative quantity!!!!**

Consider a  
Temperature profile  
with

Lapse rate

$$\Gamma = 6^{\circ}\text{C km}^{-1}$$

$$\Gamma = -\frac{dT}{dz}$$



$$p = 1000 \text{ mb} \implies \theta = T$$

For  $p < 1000 \text{ mb} \implies \theta > T$  ← Adiabatic compression  
to lower the parcel

$p > 1000 \text{ mb} \implies \theta < T$  ← Adiabatic expansion to  
rise the parcel

## Considering Water vapor

$$\theta = T \left( \frac{p_0}{p} \right)^{\frac{R}{c_p}} \quad \frac{R}{c_p} \quad ?? \quad \text{Considering water vapor}$$

Specific heat  
for moist air

$$c_p = (1 - q_v)c_{pd} + q_v c_{pv} \approx c_{pd} (1 + 0.87q_v)$$

$d$  For dry air

$v$  For water vapor

$q_v$  Specific humidity

And knowing  
(from Ch. 1)

$$R = R_d (1 + 0.608q_v)$$

$$\frac{R}{c_p} = \frac{R_d}{c_{pd}} \left( \frac{1 + 0.608q_v}{1 + 0.87q_v} \right) \approx \frac{R_d}{c_{pd}} (1 - 0.26q_v)$$

$$\Rightarrow \theta = T \left( \frac{p_0}{p} \right)^{\frac{R_d (1 - 0.26q_v)}{c_{pd}}}$$

Potential Temperature for Moist Air

The difference between dry-air and moist air potential temperature is generally less than 0.1 degree.

Adiabatic expansion or compression of moisture air can be treated as if it were dry air

**Important:**  $\theta$  is not conserved if a change of phase occurs

Liquid to vapor  
solid to Liquid

### Virtual Temperature

$$T_v$$

the temperature a parcel which contains no moisture would have to equal the density of a parcel at a specific temperature and humidity.

### Virtual Potential Temperature

$$\theta_v$$

temperature a parcel at a specific pressure level and virtual temperature would have if it were lowered or raised to 1000 mb.

### Virtual Potential Temperature

Neglect the dependence on water vapor from the exponent, and replace temperature by virtual temperature

$$\theta = T \left( \frac{p_0}{p} \right)^{\frac{R_d(1-0.26q_v)}{c_{pd}}} \longrightarrow \theta_v = T_v \left( \frac{p_0}{p} \right)^{\frac{R_d}{c_{pd}}}$$

## Adiabatic Ascent of a Parcel

$$T \downarrow \quad \theta \text{ constant}$$

Rate of decrease of temperature with height (first law, enthalpy form)

$$c_p dT = v dp$$

First law for adiabatic process

If there are no large vertical accelerations (hydrostatic relation applies)

$$-gdz = v dp \longrightarrow c_p dT = -gdz$$

$$\Gamma = -\frac{dT}{dz}$$

$$\Gamma_d = \frac{g}{c_{pd}} \approx 9.8^\circ \text{C km}^{-1}$$

Lapse rate for dry air

For moist air

$$\Gamma = \frac{g}{c_{pd}(1+0.87q_v)}$$